Sound in Multiples: Synchrony and Interaction Design using Coupled-Oscillator Networks

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ABSTRACT

Systems of coupled-oscillators can be employed in a variety of algorithmic settings to explore the self-organizing dynamics of synchronization. In the realm of audio-visual generation, coupled oscillator networks can be usefully applied to musical content related to sound synthesis, rhythmic generation, and compositional design. By formulating different models of these generative dynamical systems, I outline different methodologies from which to generate sound from collections of interacting oscillators and discuss how their rich, non-linear dynamics can be exploited in the context of sound-based art. A summary of these mathematical models are discussed and a range of applications are proposed in which they may be useful in producing and analyzing sound. I discuss these models in relationship to one of my own kinetic sound sculptures to analyze to what extent they can be used to characterize synchrony as an analytical tool.

1. INTRODUCTION

Coupled Oscillators networks are dynamical systems that describe how ensembles of interacting elements are able to self-organize and synchronize over time. In terms of sensory perception, they have been examined in a wide range of fields including those related to rhythmic entrainment, biomusicology, psychoacoustics, signal processing, and generative music [1, 2]. In the field of computer music, there have been a plethora of synthesis techniques that attempt to generate interactive and collective phenomena. These include techniques related to additive and granular synthesis, microsound, swarm models, texture synthesis, physical modeling synthesis, and statistical signal processing [3–5]. Previous work in coupled oscillators as a generative musical devices has been explored by Lambert where he looks at coupled oscillators as a "stigmergic" model, producing complex output through an audience's interaction with a system of coupled Van der Poll oscillators [6]. Operating within a similar territory, this paper proposes several generative paradigms to create sound in different synthesis and rhythmic schemes. Lastly, I describe one of my own kinetic sound sculptures that was inspired from the system dynamics of a specific coupled oscillator model.

2. MATHEMATICAL DESCRIPTION

Coupled oscillators are a broad category of interacting dynamical systems that describe a wide range of natural phenomena such as firefly synchronization, pace maker cells, neural networks, and cricket chirping models [7, 8]. One of the most basic coupled oscillator models is known as the Kuramoto model [9]. In this formulation, the governing equation for each oscillator's phase is shown for the ensemble in Equation (1)

$$\dot{\phi_i} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} sin(\phi_j - \phi_i - \alpha_o) \qquad (1)$$

where ϕ_i is the phase of the i_{th} oscillator and $\dot{\phi_i}$ is the derivative of phase with respect to time. ω_i is the intrinsic frequency of the oscillator, i, in a population of N oscillators. K is the coupling factor and the $sin(\phi_j - \phi_i)$ term is the phase response function that determines the interaction between each oscillator and the group. We can add a phase offset or "frustration" parameter α_o in the phase response function to force oscillators into different phase orientations or to account for a time delay in the model.

As a visual description, it's useful to describe the system by the movement of a "swarm of points" moving about a circle, each point representing one oscillator with its own intrinsic frequency drawn from a probability distribution, $g(\omega)$ (which is generally taken to be a unimodal gaussian distribution).

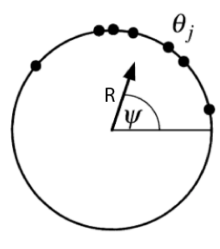

Figure 1. "Ensemble of coupled oscillators represented in a circle map as a "swarm of points" moving about a circle [7].

Depending on the $g(\omega)$ from which the oscillators are drawn, Kuramoto was able to show that in the limit as N goes to ∞, the critical coupling, K_c, will define the point at the which the system will undergo a phase state transition characterized by collective synchrony. This critical coupling, K_c is shown in Equation (2)

$$K_c = \frac{2}{\pi g(0)} \tag{2}$$

where $g(0)$ is the mean of the distribution of initialized intrinsic frequencies in the set of ω_i. If $K > K_c$ the oscillators' phases will begin to spontaneously align and the system state can be said to be characterized by synchrony.

We can extract the complex order parameters, R (phase coherence) and ψ (average phase) to solve for the system in the limit as N goes to ∞. This modifies the governing equation to be in terms of a mean-field approximation of the oscillators' phases: each oscillator is no longer beholden to the phase of every other oscillator but is coupled to the ensemble's summed, average phase. This is shown in Equation (3).

$$Re^{j\psi} = \frac{1}{N}\sum_{i=1}^{N} e^{j\psi_i} \tag{3}$$

The phase coherence R is a good indication of the synchrony of the system at large: when $R = 1$ the system exhibits complete synchrony (all phases are aligned) and when $R = 0$, the oscillators are desynchronized (points are simply running around the circle at their own intrinsic frequency, ω_i). Applying these complex order parameters to Equation (1), we form Equation (4).

$$\dot{\phi}_{\mathrm{i}} = \omega_i + KR\sum_{j=1}^{N} sin(\psi - \phi_i) \tag{4}$$

We can add an external forcing term by adding another term with a different phase response function, $\Lambda_e(\phi_i)$ as shown in Equation (5).

$$\dot{\phi}_i = \omega_i + \Lambda_e(\phi_i) + KR\sin(\psi - \phi_i) \tag{5}$$

Now the system equations demonstrate a trade-off between frequency alignment by external forcing and phase alignment by the attractive coupling as a function of their phase response curves. We can choose $\Lambda(\phi)$ to be from any distribution but certain functions are associated with different system behavior. For example, if we let Λ be a "sawtooth interaction function" [10], we can force the oscillators into a "incoherent state" where all oscillators will settle on the same frequency but with a constant phase offset, α_o, as seen in Equation (1). Depending on N, this will space out the oscillators to have a constant phase offset, $0 < \phi_c < 2\pi$.

More complex behavior can emerge when we let ω_i, K, N and $\Lambda(\phi)$ of Equation (5) become a function of time as well. Additionally, even more complex behavior arises when we let K take on different values between different micro-ensembles of coupled oscillators.

The complex order parameters are simply one way to evaluate the group synchrony of the system. Frank and Richardson's "cluster phase method" uses the complex order parameters to derive another degree of synchronization in multi-variate time series [11]. These have implications in different sonification, synthesis and rhythmic schemes that result from the aforementioned generative model.

3. COUPLED OSCILLATORS AS GENERATIVE SONIC DEVICES

Using this coupled oscillator model, we can extend these different parameters and states to synthesize sound on a continuum of collective rhythms both at the beat and sample level. As a general paradigm, rather than solving these N^{th} order equations analytically (which computationally can become intractable rather quickly), we can generate the system output using numerical analysis and employ it to generate sound in several different ways. As such, we can modify the rate at which the system is generated and map the output to sonic parameters in different perceptual time scales. This is the crucial link that maps a theoretical mathematical system to the sensory phenomena of auditory processing. For this end, this approach can be looked at as a sonification of the data that operates on a temporal spectrum.

3.1 Sound synthesis with Coupled Oscillators

3.1.1 Additive Synthesis

Because these non-linear oscillators trace out sinusoidal trajectories, the most basic synthesis method would be to simply treat the system as an oscillator bank where each oscillator's instantaneous phase is a signal amplitude at an audio rate. As the system begins to self-organize and phase-align, their collective entrainment would be perceived as a collection of sine waves of different initial frequencies emerging to a single frequency over time. As the coupling coefficient is increased to reach K_c, Fig. 2 shows the power spectrum of a group of oscillators becoming entrained to the center frequency of a gaussian distribution of oscillators from 0 to 5 kHz. Here we can see how oscillators with intrinsic frequencies near the center of frequency distribution are recruited (or entrained) first whereas oscillators near the tails of the distribution take longer to synchronize to the mean frequency.

We can also replace the external forcing in Equation (5) with the frequency content of audio that could drives the individual oscillators. In this synthesis model, the oscillators could act as a N_{th} order filter bank with center frequencies determined by their assignable intrinsic frequencies. This differs from a phase vocoder model insofar as the center frequencies of the filter bank are not fixed in frequency but are coupled according to some schema and therefore allowed to deviate by some amount.

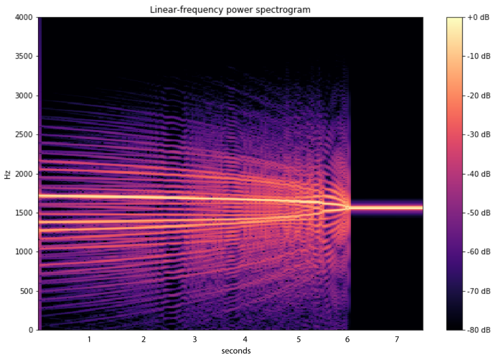

Figure 2. Ensemble of 100 sinusoidal oscillators becoming entrained to a frequency at the center of the distribution.

3.1.2 Spectral Processing

Other more complex synthesis techniques can be derived from extracting instantaneous phase from an input audio file using instantaneous frequency estimation techniques where the signal can be decomposed into a collection of instantaneous phases (via a Hilbert Transform and phase unwrapping). We can apply coupling between the instantaneous phases using coupled oscillator dynamics to perform transformations on the temporal or spectral information. For example using a FFT interpretation of the phase vocoder model, we can divide the time-varying signal into several spectral bands and–after unwrapping each channel's instantaneous phase–apply band-limited coupled oscillator networks to modulate their instantaneous phase forcing them to become entrained to a center frequency within the spectral band over time. Because the critical coupling of Equation (2) is a function of the intrinsic frequency distribution set by $g(\omega)$, we can populate these spectral regions of the input spectrum with oscillators drawn from a gaussian distributions with μ centered around the FFT bin center frequency. This has the effect of encouraging oscillator synchronization within the channel-dependent (band-limited, constant-Q) region. In this synthesis scheme, the external forcing function, $\Lambda_e(\phi_i)$, is passed the instantaneous phases extracted from each of the spectral bands by the FFT. An example is shown in Fig. 3 where a flute playing a major scale is resynthesized using the aforementioned method. This example makes use of 130 oscillators split into 10 coupled groups where coupling is increased over the duration of the sound file ultimately resulting in full synchrony per band-limited group.

Ultimately, this coupled-oscillator phase vocoder model would allow the frequency content of an input audio signal to modulate and synchronize the frequency content (or spectral entrainment) of the source sound. Sounds that are characterized by spectra that conforms to certain harmonic relationships could force the coupled oscillators into different periodic or synchronous states. Clearly, because this method utilizes phase vocoding analysis, it would work best with analysis techniques that prioritize horizontal phase coherence over vertical phase coherence.

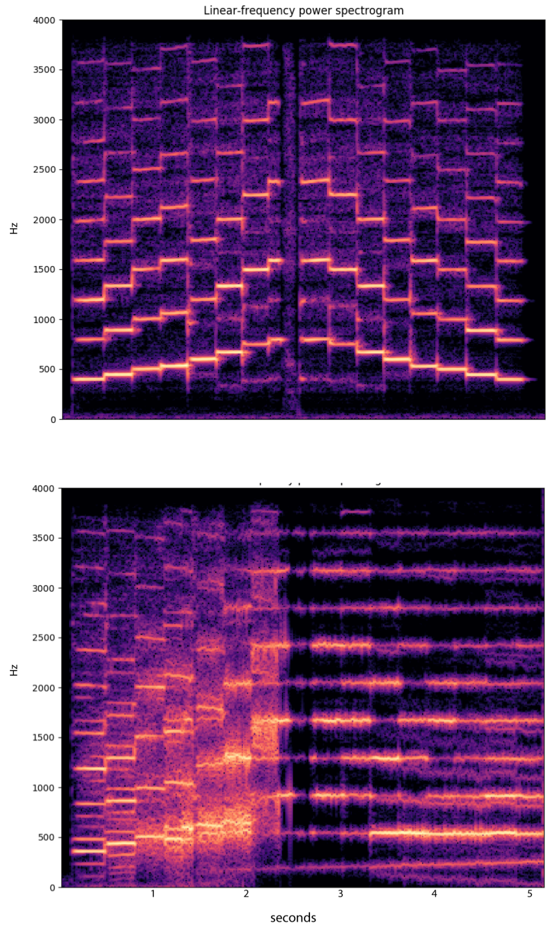

Figure 3. Spectrum of flute playing a major scale (top). Phase vocoder coupled oscillator resynthesis using ten coupled oscillator networks (bottom).

3.2 Rhythmic Generation: Coupled Oscillators as Control Signals

We can use the dynamics of the coupled oscillator system to control rhythmic generation or musical parameters. The idea of synchronization lends itself well to many aesthetic ideas of minimalist and procedural music where musical parameters are slowly modulated over time.

If we set the coupled oscillator ensembles to be iterated at a rate that is well below a sampling rate suitable to audio synthesis, we can use Equation (5) to trigger audio events when the instantaneous phase of each oscillator ϕ_i encounters a zero-crossing. To accomplish this, we can trigger an "audio event" using the basic sonification scheme detailed in Equation (6) below.

$$\text{audio event}(\phi_i) = \begin{cases} 1, & \text{if } \phi_{i-1} < \phi_i \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Therefore, as each oscillator completes one cycle (crosses the zero-point of the circle), they trigger a sound such as the playback of a sample. The system generates complex rhythmic behavior when different groups of oscillators take on different coupling coefficients to form microensembles that are locally coupled. When the system parameters are modulated over time, the system can be forced into different polyrhythmic relationships that converge and devolve over time.

This could also be meaningfully applied to musical forms by allowing the instantaneous phase of each oscillator to control the position of a virtual "playback" head of a rhythmic figure to create complex temporal canons that can be brought together in temporal unison by adjusting the coupling coefficients over time. Similarly in the realm of synthesis, we could allow the instantaneous phase to control the playback (or the index of a buffer) of a sampled sound file in a buffer. In this paradigm, the system produces control signals to modulate parameters of a piece of music.

4. SONIC PHENOMENA AND COUPLED OSCILLATORS

If these mathematical models are sufficiently generalizable and applicable to musical analysis, they can describe and generate a plethora of meaningful musical techniques with examples taken from contemporary music composition and sound art. Perhaps the most axiomatic example demonstrating collective perceptual entrainment is Györgi Ligeti's *Poème Symponique"* (1962) for 100 metronomes [12]. In this piece one-hundred metronomes are pre-wound, set to different tempos, and then triggered en masse. As each metronome comes to rest at different times, the dynamics of the ensemble at large are well modeled using a uncoupled oscillator model where each metronome is set to a different ω_i. As different auditory streams of periodic rhythm are presented concurrently, the listener latches onto different frames of temporal reference where a sense of beat (induction) emerges from their competing periodic stimuli. Modifying Ligeti's original piece by coupling the metronomes by placing them onto a low-friction surface such as a table with wheels, the mechanical movements of the pendulums will begin to couple their swinging motion to one another. If coupling is sufficiently strong, the metronomes will become phase-aligned to tick at a mean frequency [13].

The simultaneity of periodic rhythms characterized by *Poème Symphonique* can also be well applied to the analysis of acoustic crowd dynamics where researchers have used coupled oscillator models with spatial mean-field coupling to account the physics of crowd applause [14]. This acoustic phenomena bears resemblance to many stochastic generative methods that are capable of modeling the sound of natural phenomena (e.g. rain, hail, wind, etc.). However, the potential of the system to be controlled to self-organize over time might allow for interesting forms of collective synchrony that emerge amidst the dense acoustic textures characterized by nature. In terms of acoustic signalling in animal populations, coupled oscillator systems have been used to describe many different forms of biomusicological phenomena particularly those related to chorusing and stridulation [1, 15, 16]. Using research from these biomusicological models, generative chorusing synthesis that incorporate coupled oscillator synchronization methods could be an interesting avenue of exploration in sound generation and user interface design.

4.0.1 Compositional Techniques

In terms of music analysis and composition, coupled oscillator dynamics of synchrony can be thought of as a temporal canon in which different fugal patterns are stretched and compressed over time to conform to a governing temporal duration. In the minimalist genre, the rhythmic "phasing" effect in Steve Reich's music (e.g. "Clapping Music", "Come Out", "Piano Phase") could be approximated by a coupled oscillator model that converges in and out of synchrony. "Phasing" could be accomplished by setting an ensemble of oscillators with different initial phases but the same intrinsic frequencies and phase-aligning them over time. Reich himself has intuited that in this compositional technique, "[t]he listener becomes aware of one pattern in the music which may open his ear to another, and another, all sounding simultaneously and in the ongoing overall texture of sounds." [17]. His formulation of pattern as rhythm reinforces similar perceptual notions of Ligeti's Poème Symphonique insofar as that the listener has access to simultaneous layers of competing perceptual information and that auditory feedback allows certain phenomena to take precedence over others.

Lastly in the field of sound-based art, several contemporary artists have experimented with auditory phenomena that is well-modeled by coupled oscillator systems. These include works by Zimoun, Pei Lang, and Céleste Boursier-Mougenot [18]. These artists are known for their use of multiples of sound objects set in repetitive motion to create large masses of sound from simple additive means. For instance, Zimoun's installation-based work employs hundreds of kinetic objects to construct complex sound masses in physical environments. Taken to the extreme, these sound sculptures make use of a material-oriented additive synthesis that could be approximated by dynamical system models.

5. MODELLING SYNCHRONY THROUGH SCULPTURAL FORM: HIVEMIND

The rich musical dynamics inherent in coupled oscillator networks have inspired my own sonic investigations in an attempt to experiment with how to exploit these systems in physical, sculptural form. Much of my understandings of coupled oscillator dynamics in sound have been through the development of computational models that have allowed me to interact with this dynamical system through mathematical analysis and numerical analysis [1]. For these pieces, I've written programs to explore numerical analysis (python), user interaction in real-time (SuperCollider), and performance based programs (CHuCK) to allow me experiment with the system behavior under different parameterizations. This repository also hosts several synthesis implementations mentioned in section 3.1.

5.1 Audio-visual Resonance: "Hivemind"

Figure 4. "HiveMind" at Pioneer Works Brooklyn,NY. video: https://vimeo.com/127874298

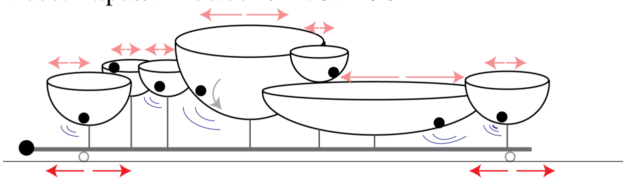

Figure 5. Kinetics of audio-visual resonance of "HiveMind" sound sculpture: clay bowls as driven coupled oscillators

"Hivemind" explores the sonic potentials of ceramics by revealing the pitched resonance of porcelain bowls using a coupled-oscillator mechanical system. Two reciprocating platforms are populated with over 300 clay vessels with marbles placed about the inner bowls. By modulating the speed of the applied pushing motion (see Fig. 5), this piece surveys the acoustic potential of ceramic as material by exposing the audio-visual "resonance" of different bowls. When this pushing motion matches the natural rotational frequency of the bowl's topography, the marble begins to rotate and loop with more velocity thereby amplifying the characteristic resonance of the bowl. Because each bowl contains a different resonant frequency, clusters of similarly-sized bowls can be amplified to create slowly-changing bell-like sonorities. The pushing motion of the two platforms drive the system into different dynamic states to form a time-based composition of audio-visual resonance.

[1] for more information, see my code repository: https://bitbucket.org/no_lem/kura-python/

From the perspective of coupled oscillation, the reciprocating platform can be thought of as a type time-dependent external forcing factor, $\Lambda_e(\phi_i, t)$ from Equation (5) that drives the system. Because the marbles' motion are beholden to this external force, each bowl can be looked at as a resonant filter at some center frequency determined by their shape. The input to these filters is simply the pushing motion by the reciprocating platform and their audible output is the sum of their (damped) oscillations. Even though the individual marbles are not explicitly coupled to one another, they resonate in concert with the frequency (and amplitude) of the external sinusoidal pushing force. To explicitly couple the oscillators, one would have to resort to a different physical implementation that would allow the instantaneous phase of each physical object to interact with the others.

6. CONCLUSIONS

This paper looked at the extent to which coupled oscillators can be useful to describe a wide range of musical phenomena by demonstrating several ways in which they model synchronous auditory phenomena. There's still much territory to be explored in this area of applied musical research. For instance, this paper only looked at one such synchronization scheme–the Kuramoto model–to describe a type of self-organization. There are several other synchronization models (pulse-coupling, sync and swarm models, Van Der Poll oscillators, etc.) that could be exploited in the context of art and music generation. Similarly, this paper only briefly mentioned several applications related to digital signal processing, rhythmic generation, or music perception. One particularly promising area of research is neural resonance theory in the context of beat induction and meter perception as posed by Large [2]. As an outgrowth of dynamic attending theory, his canonical model accounts for the entrainment of endogenous cortical rhythms from the acoustic rhythms of the external world [19]. More importantly, his canonical model is derived from a coupled-oscillator model of dynamical systems. Future research learning how to integrate these notions of perceived beat and rhythm into different generative models would be well served in the area of music creation and sound art.

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